

Science is a lively social activity, with many claims being lively debated. What about mathematics? The cliché about mathematicians being poor at managing social relations is quite strong and widespread. One of the most famous joke on the topic goes like this:

Question: How can you spot an extrovert mathematician?

Answer: He looks at YOUR shoes when he talks to you.

Is mathematics "less social" than other academic disciplines? Some support for a 'yes' answer can be found in a recent piece of news. A famous mathematician, Nelson, had claimed to give a proof of a rather surprising proposition: "Peano Arithmetic is inconsistent." Two other famous mathematicians, Tao and Tausk, said the proof included one specific mistake, which they spelled out. Nelson's reaction was: "Ah, you're right. So I have not proven that Peano Arithmetics is inconsistent". End of the story. No fight, no disagreement, no formation of alternative schools of thoughts, no playing with how to interpret this or that claim. Just plain boring consensus.

$$\forall x S(x) \neq 0$$

$$(\forall x (\forall y (S(x) == S(y) \Rightarrow x == y)))$$

$$\forall x x + 0 == x$$

$$\forall x (\forall y x + S(y) == S(x + y))$$

$$\forall x x 0 == 0$$

$$\forall x (\forall y x S(y) == y x + x)$$

$$A(0) \Rightarrow ((\forall x (A(x) \Rightarrow A(S(x)))) \Rightarrow (\forall y A(y)))$$

(These are the axioms that Nelson claimed were inconsistent. They are supposed to express central propositions true of our system of natural numbers with addition. They are used to prove things about an object that is central in many cultures.)

Mathematics is full of that: easily achieved consensus. Everybody agrees. No debate, and yet, the consensus is not socially induced in any standard way.

In a recent blog post, [The \(in\)consistency of PA and consensus in mathematics](#), Catarina Novaes takes this as a nice illustration of some points made by philosopher Jody Azzouni, who argues that [mathematics is unique as a social practice](#).

1. While mathematicians do make mistakes, once the mistakes are spotted, consensus is easily achieved.
2. Consensus is not achieved through coercion or because of some social interests; there is a shared genuine conviction that is acquired independently of the opinion of others (the arguments might be displayed by others, but the opinion is formed only in view of the arguments themselves, not in view of who has formulated the arguments).
3. Internalisation does play a role, since mathematics is learned, but "homogeneous indoctrination" or "internalisation of social standards" is not an explanation of homogeneous opinion on a proof.

The third point needs further explanations. Azzouni's argument is that, in mathematics, there has been no "drift" through small variations in interpretations. The social standard of what constitutes a proof has remained stable across time and so has the social practice of proving. This contrasts with the internalisation of, say, religious beliefs, where a different opinion can always lead to an alternative religion. Azzouni notes that « mistakes in mathematics are common, and yet mathematical culture doesn't splinter because of them, or for any other reason (for that matter); that is, permanent competing practices don't arise as they can with other socially-constrained practices. This makes mathematics (sociologically speaking) very odd. »

I think that the research question is well put: doing mathematics is a social practice, but it differs in important ways from most other social practices. Mathematics is learned, communication plays a crucial role in its maintenance and development, mathematicians form a distinct community with its institutions ... and there is a strong, puzzling, homogeneity of opinion. (I am not convinced that standards of proofs did not evolve and that debates in mathematics had little impact on the practice itself, as Azzouni suggests; but it remains that the ease through which uniformity of opinion is achieved is puzzling).

What causes this homogeneity? It is mainly when one thinks of sociology as limited to analyses of power relations — of how groups of people impose beliefs and practices on others, that social investigation is deemed irrelevant (giving way to Platonism or psychologism). If, by contrast, social investigation means looking at all the factors that might produce consensus and homogeneity of opinion, then a social and naturalistic investigation can be carried on. Thus the slogan: when facing the specificity of mathematics, don't put sociology out; but put psychology in. In particular, we can hope that psychologists will have something to say about why mistakes are so easily acknowledged. What happens, for instance, with the confirmation bias? Why are evaluations of proofs generating no controversies, while evaluations of scientific arguments do?